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UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2024

UPHYMAJ11001-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: 1×5 = 5

(a) Determine the constant 'a' so that the vector $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal.

(b) A scalar field is defined as $\phi(x, y, z) = 4yz^3 + 3xyz - z^2 + 2$. Find out $\phi(1, -1, 2)$.

(c) A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ [t = time]. Find out its velocity at any time.

(d) Find out $\vec{\nabla}\phi$ if $\phi = \ln|\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(e) Find out the square of the element of arc length ds^2 in cylindrical co-ordinates (ρ, ϕ, z).

(f) Find out the order and degree of the equation $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 8 = 0$.

(g) Find out the value of $\oiint_S \frac{\vec{r}\cdot\overrightarrow{dS}}{r^3}$ where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin.

(h) The point of application of a force $\vec{F} = 5\hat{i} + 10\hat{j} + 15\hat{k}$ is displaced from the point (1, 0, 3) to the point (4, –1, –6). Find out the work done by the force.

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. If $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{F}\cdot\hat{n}\,dS$, where S is the surface of the cube bounded by $(x=0,\ x=1)$, $(y=0,\ y=1)$, $(z=0,\ z=1)$ and $\hat{n}$ is a unit vector normal to the surface element dS. 5

3. Express curl $\vec{A} = \vec{\nabla}\times\vec{A}$ in the spherical polar (orthogonal) co-ordinate system. 5

4. Solve the differential equation: 5

$$\sin x\frac{dy}{dx} - y\cos x + y^2 = 0$$

5. If $\vec{A} = \hat{i}(xy + y^2) + \hat{j}(2y - 3xy)$, determine $\oint \vec{A}\cdot\vec{dr}$ along the triangle ΔPQR, where the position co-ordinates of P, Q and R are (0, 0), (2, 0) and (2, 1), respectively. 5

6. (a) Determine $\vec{\nabla}\cdot(r^n\vec{r})$. What is the value of n for which $r^n\vec{r}$ is a solenoidal vector? 2+1

(b) Determine the unit vector perpendicular to the plane $x^2 + 3y^2 + 2z^2 = 6$ at the point (2, 0, 1). 2

GROUP-C

Answer any *two* questions from the following $10\times2 = 20$

7. Express in cylindrical co-ordinate system the following quantities: $2\frac{1}{2}\times4 = 10$

(i) $\vec{\nabla}\phi$ (ii) $\vec{\nabla}\cdot\vec{A}$

(iii) $\vec{\nabla}\times\vec{A}$ (iv) $\nabla^2\phi$

8. Solve the following initial-value problems. $5\times2 = 10$

(a) $2y'' + 5y' + 3y = 0$, $y(0) = 3$, $y'(0) = -4$

(b) $y'' + 16y = 0$, $y(\pi/4) = -3$, $y'(\pi/4) = 4$

9. (a) Find out the directional derivative of ϕ, where $\phi(x, y, z) = x^2y^2z^2$, at the point $(1, -1, 2)$ along the vector $\hat{i} - 2\hat{j} + 2\hat{k}$. 4

(b) If $\vec{v} = \vec{\omega}\times\vec{r}$, where $\vec{\omega}$ is a constant vector and $\vec{r}$ is the position vector, then show that $\vec{\omega} = \frac{1}{2}(\vec{\nabla}\times\vec{v})$. 4

(c) Evaluate $\vec{\nabla}\times\left(\frac{\vec{r}}{r^2}\right)$. 2

10.(a) The force $\vec{F} = (-3\hat{i} + \hat{j} + 5\hat{k})$ acts at the point $(7\hat{i} + 3\hat{j} + \hat{k})$. Calculate the torque about the point (0, 10, 0). 4

(b) Show that the closed path line integral of the vector $\vec{A} = -y\hat{i} + x\hat{j}$ is equal to twice the area enclosed by the closed path. 3

(c) Prove that $\vec{\nabla}(\vec{A}\cdot\vec{r}) = \vec{A}$, where $\vec{A}$ is a constant vector and $\vec{r}$ is the position vector. 3

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